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APPROXIMATE FEEDBACK LINEARIZATION OF AN AIR-BREATHING HYPERSONIC VEHICLE (PREPRINT)

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Approximate Feedback Linearization of an Air-breathing Hypersonic Vehicle

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This paper describes the design of a nonlinear control law for an air-breathing hypersonic vehicle. The model of interest includes flexibility effects and intricate couplings between the engine dynamics and flight dynamics. To overcome the analytical intractability of this model, a nominal control-oriented model is constructed for the purpose of feedback control design. Analysis performed on the nominal model reveals the presence of unstable zero dynamics with respect to the output to be controlled, namely altitude and velocity. By neglecting certain weaker couplings and resorting to dynamic extension at the input side, a simplified nominal model with full vector relative degree with respect to the regulated output is obtained. Standard dynamic inversion can then be applied to the simplified nominal model, and this results in approximate linearization of the nominal model. Finally, a robust outer loop control is designed using LQR with integral augmentation in a model reference scheme. Simulation results are provided to demonstrate that the approximate feedback linearization approach achieves excellent tracking performance on the truth model for two choices of the system output. Finally, a brief case study is presented to qualitatively demonstrate the robustness of the design to parameter variations.

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Nomenclature, States and Inputs

State		Input	
V_t	Vehicle velocity	δ_e	Control surface deflection
α	Angle of attack	Φ	Fuel to air ratio
Q	Pitch rate	A_d	Diffuser area ratio
h	Altitude	x_d	Cowl lip position
θ	Pitch angle		
η_i	ith Generalized elastic coordinate		
$\dot{\eta}_i$	Time derivative of η_i		

Table 1. States and inputs for the model

I. Introduction

Air-breathing hypersonic vehicles may eventually allow dramatic reductions in flight times for both commercial and military applications. Direct access to earth orbit without the use of separate boosting stages may also become possible as scramjet powered aircraft enter service. While numerous challenges remain, recent successes with NASA's X-43a and renewed research activities throughout the aerospace community suggest that this technology is well on its way to assuming a role in the next generation of aviation.

The design of controllers for air-breathing hypersonic vehicles requires the controls engineer to overcome strong couplings between the propulsive and aerodynamic effects while also addressing the significant flexibility associated with the slender geometries required for these aircraft.[2,14,15] As a result, all the control schemes proposed to date have been limited in their scope to the longitudinal dynamics of hypersonic vehicles. A wide range of control laws have been developed for linearized versions of hypersonic models,[1,12,14–16] while a few have also attempted to incorporate guidance.[5,17] Tournes et al. employed a nonlinear variable structure control approach in Ref. 18, while several other nonlinear control approaches have been completed on nearly non-flexible models.[6,11,19,20]

This paper deals with nonlinear control of the model of a flexible air-breathing hypersonic vehicle developed by Bolender and Doman,[2] which employs compressible flow theory to obtain a more complex and potentially more complete model than those developed in earlier, similar efforts.[3,4] Following the successful design of linear controllers[8] for the model considered in this paper, the next logical step is the synthesis of a nonlinear control law for this novel plant. In this initial study, a traditional feedback linearization approach will be pursued.

Intricate interactions between the structural, aerodynamic, and propulsion system equations result in exceedingly complex expressions for the aerodynamic forces and coupling terms between the flexible states and the other equations of motion. When written in a form suitable for control, these equations are clearly analytically untractable, even with the aid of a computer algebra system. The intractable model will be referred to as the truth model and will be used solely for simulation and control design verification.

The first step in the control design process is the creation of a nominal model which approximates the behavior of the truth model with reduced complexity. The outputs to be controlled are chosen as the vehicle velocity and altitude. The control inputs are the elevator deflection and fuel to air ratio. The resulting non-linear nominal model is at first glance very similar to those given in previous works.[19,20] However, as opposed to the models considered in Refs. 19, 20, the addition of flexible states, increased complexity of the engine model, and additional parasitic couplings between the control effectors and the aerodynamic forces create an unstable zero dynamics and drastically increase the complexity of the vehicle's governing equations.

Rather than attempting to design control directly for this nonminimum-phase system, the approach of approximate feedback linearization is employed.[9] The flexible states and a set of weak couplings are removed from the nominal model to create a simplified nominal model which can be shown to posses full vector relative degree with appropriate dynamic extension. Thus, a straight-forward design of a feedback linearization control law can be completed for this simplified nominal model. An outer loop controller is then designed using linear quadratic regulator (LQR) methods with integral augmentation and model reference

control. Simulation studies reveal that, despite the simplified model of the plant dynamics, this control law performs admirably on both the nominal and truth models.

The paper is organized as follows: section II describes the construction of the nominal model and the system identification process that is employed to match it to truth model data. In section III, the simplified nominal model is presented. The control law design is described in section IV, and simulation results for the truth model are given in section V along with simulation results for small plant parameter variations. Finally, an alternative choice of outputs and the corresponding simulation results are given in section VI, while section VII concludes the document with a brief summary of the results.

II. Nominal Model

In this section, we derive a control-oriented version of the model of the longitudinal dynamics of an air-breathing hypersonic vehicle developed in Ref. 2. The simplification is necessary to overcome the analytical intractability of the original model, which will be henceforth referred to as the *truth model*. For the sake of simplicity, the control oriented model will be referred to as the *nominal model*. After the synthesis of the model is completed, its suitability for control design is evaluated.

A. Model Development

The nonlinear longitudinal equations of motion for the model developed by Bolender and Doman[2] are given as

$$\dot{V}_t = \frac{1}{m}(T\cos\alpha - D) - g\sin(\theta-\alpha) \tag{1}$$

$$\begin{aligned}
\dot{\alpha} &= \frac{1}{mV_t}(-T\sin\alpha - L) + Q + \frac{g}{V_t}\cos(\theta-\alpha) \\
I_{yy}\dot{Q} &= M + \tilde{\psi}_1\ddot{\eta}_1 + \tilde{\psi}_2\ddot{\eta}_2 \\
\dot{h} &= V_t\sin(\theta-\alpha) \\
\dot{\theta} &= Q \\
k_1\ddot{\eta}_1 &= -2\zeta_1\omega_1\dot{\eta}_1 - \omega_1^2\eta_1 + N_1 - \tilde{\psi}_1\frac{M}{I_{yy}} - \frac{\tilde{\psi}_1\tilde{\psi}_2\ddot{\eta}_2}{I_{yy}} \\
k_2\ddot{\eta}_2 &= -2\zeta_2\omega_2\dot{\eta}_2 - \omega_2^2\eta_2 + N_2 - \tilde{\psi}_2\frac{M}{I_{yy}} - \frac{\tilde{\psi}_2\tilde{\psi}_1\ddot{\eta}_1}{I_{yy}},
\end{aligned} \tag{2}$$

where the system state $x_n \in \mathbb{R}^9$ is comprised of the nine state variables V_t, α, Q, h, θ, η_1, η_2, $\dot{\eta}_1$, and $\dot{\eta}_2$ which are defined in Table 1 along with the four control inputs. This formulation of the system differs slightly from that given in Ref. 2 in that the temperature change across the combustor input ΔT_0 has been replaced with the functionally equivalent control input Φ, the fuel to air ratio. The change was made since Φ is a more natural input choice for aerospace design. The outputs to be controlled are V_t and h. Since only two control inputs are required for the feedback linearization design, x_d and A_d will be fixed at the nominal values $x_d = 0$ and $A_d = 1$ for the remainder of this study. Table 2 defines several physical constants derived from the vehicle geometry, aerodynamic conditions, and assumed elastic mode shapes.[2]

Physical Constant	
m	Vehicle mass
I_{yy}	Moment of intertia
$\tilde{\psi}_i$	Constrained beam coupling constant for η_i
k_i	$1 + \frac{\tilde{\psi}_i}{I_{yy}}$
ζ_i	Damping ratio for elastic mode η_i
ω_i	Natural frequency for elastic mode η_i

Table 2. Physical constants included in the equations of motion (1) to (3)

The lift L, drag D, thrust T, pitching moment M, and the two generalized forces N_1 and N_2 are complex algebraic functions of both the system state and the inputs that must be simplified to render the model analytically tractable. While attempts to represent these force mappings with cubic-spline interpolated lookup tables were very successful,[7] these approximations are not suitable for analytical analysis with the tools of feedback linearization. Thus, drawing on earlier work done with a non-flexible vehicle,[19,20] the following approximations to these mappings will be adopted for the nominal model

$$\begin{aligned}
L &\approx \frac{1}{2}\rho V_t^2 S C_L \qquad (3)\\
D &\approx \frac{1}{2}\rho V_t^2 S C_D\\
M &\approx z_T T + \frac{1}{2}\rho V_t^2 S \bar{c}[C_{M,\alpha} + C_{M,\delta_e}]\\
N_1 &\approx N_1^{\alpha^2}\alpha^2 + N_1^{\alpha}\alpha + N_1^0\\
N_2 &\approx N_2^{\alpha^2}\alpha^2 + N_2^{\alpha}\alpha + N_2^{\delta_e}\delta_e + N_2^0.
\end{aligned}$$

The expressions for L and D are the same as those given in Ref. 20. The expression for M contains the additional term $z_T T$ to account for the pitching moment produced by the underslung scramjet engine in the model.[2] Finally, the numerical studies completed by Groves and Sigthorsson[7] support approximating the N_1 force as a quadratic of the angle of attack and the N_2 force as a quadratic of the angle of attack with a linear elevator deflection term. The Q coupling term to the moment has also been neglected, because it proved inconsequential for this vehicle. The C_* coefficients are given as follows

$$\begin{aligned}
\rho &= \rho_0 \exp\left(\frac{-(h-h_0)}{h_s}\right)\\
C_L &= C_L^{\alpha}\alpha + C_L^{\delta_e}\delta_e + C_L^0\\
C_D &= C_D^{\alpha^2}\alpha^2 + C_D^{\alpha}\alpha + C_D^{\delta_e^2}\delta_e^2 + C_D^{\delta_e}\delta_e + C_D^0\\
C_{M,\alpha} &= C_{M,\alpha}^{\alpha^2}\alpha^2 + C_{M,\alpha}^{\alpha}\alpha + C_{M,\alpha}^0\\
C_{M,\delta_e} &= c_e(\delta_e - \alpha).
\end{aligned}$$

The expressions for the C_* coefficients are similar to those in Ref. 20, with minor notation changes and the addition of elevator terms in the lift and drag coefficients. A simple exponential model for the air density ρ is also included.

The scramjet engine included in the model produces a thrust force T which depends strongly on the states h, V_t, and α, along with the input Φ. The mapping is approximately cubic in the angle of attack, while each coefficient of this polynomial is a linear function of Φ. The following approximation for T is adopted for the nominal model

$$\begin{aligned}
T &\approx C_T^{\alpha^3}\alpha^3 + C_T^{\alpha^2}\alpha^2 + C_T^{\alpha}\alpha + C_T^0\\
C_T^{\alpha^3} &= \beta_1\Phi + \beta_2\\
C_T^{\alpha^2} &= \beta_3\Phi + \beta_4\\
C_T^{\alpha} &= \beta_5\Phi + \beta_6\\
C_T^{0} &= \beta_7\Phi + \beta_8\\
\bar{q} &= \frac{1}{2}\rho V_t^2. \qquad (4)
\end{aligned}$$

The eight β_i coefficients vary with the dynamic pressure $\bar{q}$ and h. Since these values change on a much slower timescale than Φ and α, the values of β_i are assumed to be constant for control design. In implementation, the coefficients are obtained using a cubic-spline interpolated lookup table based on coefficients derived for 400 different flight conditions.

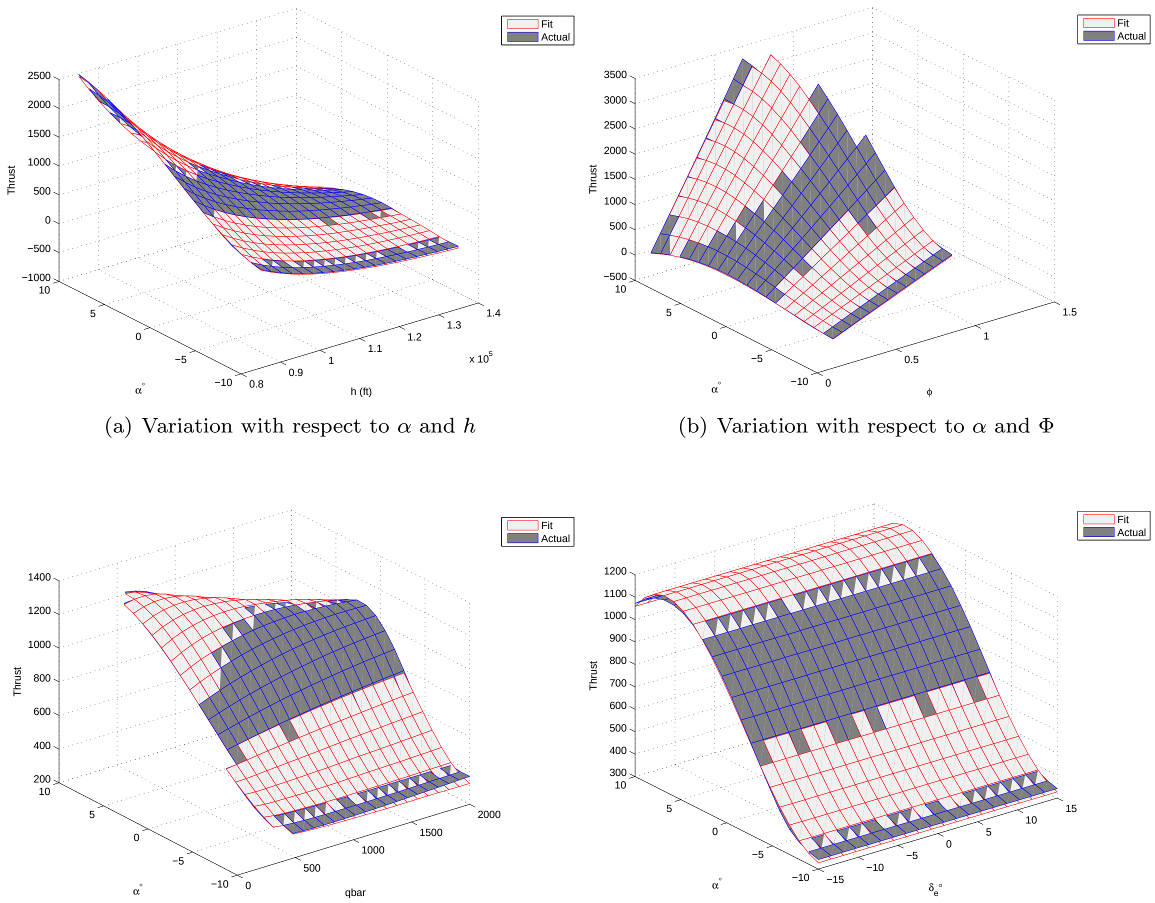
All of the force approximations given in equations (3) to (4) are linear in their fit parameters. The values for ρ_0, h_s, h_0, z_T, S, and $\bar{c}$ are known constants. The other parameters have been obtained through a least squares approach by fitting the functions to truth model data calculated over the following ranges

$$h \in [85000, 135000]$$

$$
\begin{aligned}
\Phi &\in [0.1, 1.2] \\
\delta_e &\in [-15, 15] \\
\alpha &\in [-10, 10] \\
\bar{q} &\in [500, 2000].
\end{aligned}
$$

Naturally the variation in $\bar{q}$ is obtained by appropriate variation of V_t. Even if a given force approximation does not depend on a particular state or input, that value will still be varied so as to capture its average effect. The flexible states η_i and $\dot{\eta}_i$ are set to zero during the calculation of the data for the least squares fits, since none of the force approximations depend on them. Many of the conditions on these ranges will not be physically realizable for the model. These points are simply ignored by the curve fitting algorithm.

The resulting curve fits use 3.2 million data points and capture at least the qualitative behavior of the model over the entire operating range. Figure 1 gives sample results for the thrust fit. The identification of these parameters completes the construction of the nominal model given by equations (1) to (4).



(a) Variation with respect to α and h

(b) Variation with respect to α and Φ

(c) Variation with respect to α and $\bar{q}$

(d) Variation with respect to α and δ_e

Figure 1. Thrust Curve Fitting Result for a typical trim condition. Notice that δ_e has minimal effect on the thrust, as expected.

B. Relative Degree of the Nominal Model

In this subsection, we will briefly review the basic definitions associated with the process of feedback linearization and then discuss the applicability of the technique to the nominal model. This analysis will highlight the benefits of approximating the nominal model to produce the simplified nominal model discussed in the subsequent section.

As discussed in a variety of sources,[10, 13] consider a generic multi-input multi-output (MIMO) system in which the input and output have the same dimension

$$\begin{aligned} \dot{x} &= f(x) + \sum_{i=1}^{m} g_i u_i \\ y_i &= h_i(x) \ (i = 1...m), \end{aligned}$$

with $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, and $y \in \mathbb{R}^m$. The vector fields f and g_i along with the functions h_i are assumed to be smooth in their arguments, allowing arbitrary derivatives of each to be calculated without concern for their existence. The following standard notation for the Lie derivative of the function h along the vector field f will be adopted

$$L_f h = \frac{\partial h}{\partial x} f(x).$$

The gradient of the scalar function h is interpreted as a row vector. Higher order Lie derivatives, $L_f^k h$ with $k \in \mathbb{N}$, are obtained recursively using this definition. For clarity, the notation $L_{u_i} h$ will sometimes be used to represent the Lie derivative of the function h along the vector field corresponding to the input u_i.

The analysis begins with taking successive time derivatives of each output. Define r_i such that $y_i^{r_i}$ is the first derivative of y_i that depends explicitly on one or more of the inputs u_i. This derivative can be expressed for each i as

$$y_i^{r_i} = L_f^{r_i} h_i + \sum_{k=1}^{m} L_{g_k}(L_f^{r_i - 1} h_i) u_k.$$

The decoupling matrix $A(x) : \mathbb{R}^n \to \mathbb{R}^{m \times m}$ contains the m^2 Lie derivatives which multiply the inputs in the m derivative expressions $y_i^{r_i}$. Specifically, A is given as

$$A(x) = \begin{bmatrix} L_{g_1} L_f^{r_1 - 1} h_1 & \cdots & L_{g_m} L_f^{r_1 - 1} h_1 \\ \vdots & \ddots & \vdots \\ L_{g_1} L_f^{r_m - 1} h_m & \cdots & L_{g_m} L_f^{r_m - 1} h_m \end{bmatrix}.$$

If the matrix $A(x_0)$ is nonsingular at x_0, then the system (5) is said to have vector relative degree r at x_0 where

$$r = \sum_{i=1}^{m} r_i.$$

If $r = n$ then the system is said to have full vector relative degree, and the input/output map can be rendered a set of m decoupled chains of integrators using the feedback law

$$u = -A^{-1} \begin{bmatrix} L_f^{r_1} h_1 \\ \vdots \\ L_f^{r_m} h_m \end{bmatrix} + A^{-1} v.$$

The new outer loop control $v \in \mathbb{R}^m$ contains the inputs to the decoupled integrator chains. If $r < n$ then this process can still be carried out, but $n - r$ states of the system will be rendered unobservable from the output and constitute an internal dynamics for the system under the input/output linearizing control law. An extensive discussion of these results from a differential geometric point of view can be found in Ref. 10.

Now, consider the particular case of (5) representing the nominal model of the air-breathing hypersonic vehicle. Recall that the nominal model state $x_n \in \mathbb{R}^9$ contains the state variables given in table 1. The outputs were selected as $y_1 = V_t$ and $y_2 = h$. Finally, the two control inputs are δ_e and Φ. For this input/output combination, the matrix $A(x_n)$ is singular over the entire flight envelope. In order to obtain vector relative degree for this case, the following dynamic extensions[13] are necessary

$$
\begin{aligned}
\ddot{\Phi} &= -2\zeta_3\omega_3\dot{\Phi} - \omega_3^2\Phi + \omega_3^2\Phi_c & (5)\\
\ddot{\delta}_e &= -2\zeta_4\omega_4\dot{\delta}_e - \omega_4^2\delta_e + \omega_4^2\delta_{e,c}, & (6)
\end{aligned}
$$

where we have defined a new input to the system as

$$
u = \begin{bmatrix} \Phi_c \\ \delta_{e,c} \end{bmatrix}.
$$

The values for ζ_3, ζ_4, ω_3, and ω_4 are selected to provide reasonable second order responses to these extended dynamics.

Considering this extended system with state

$$
x_{na} = \begin{bmatrix} x_n \\ \Phi \\ \dot{\Phi} \\ \delta_e \\ \dot{\delta}_e \end{bmatrix},
$$

the new matrix of Lie derivatives $A(x_{na})$ is nonsingular over the operating range of interest. Thus, the nominal model dynamically extended with (5) and (6) has a well defined vector relative degree. However, since $r = 7$, the nominal model under the feedback linearizing control law (5) will contain a six-dimensional unobservable internal dynamics. The following diffeomorphism is used to write this system in normal form

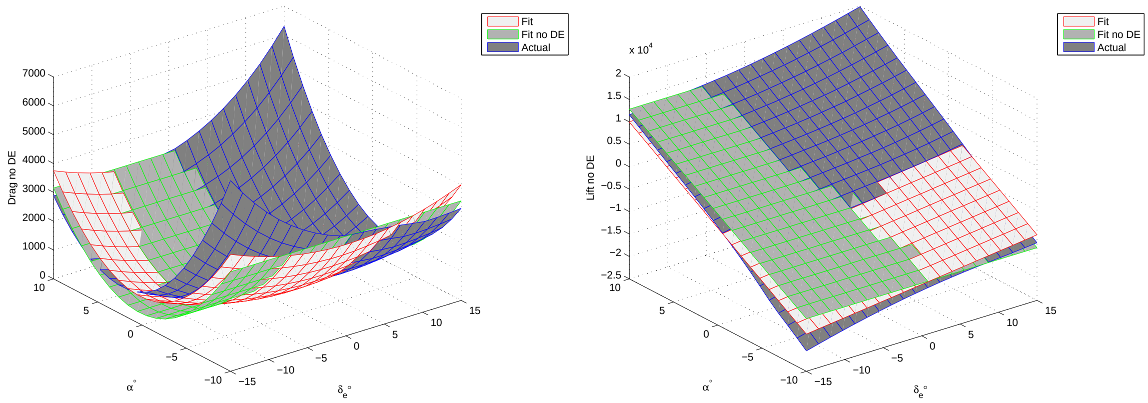
$$
\Gamma(x) = \begin{bmatrix} V_t \\ L_f V_t \\ L_f^2 V_t \\ h \\ L_f h \\ L_f^2 h \\ L_f^3 h \\ \eta_1 \\ \dot{\eta}_1 \\ \eta_2 \\ \dot{\eta}_2 \\ \theta \\ Q \end{bmatrix}.
$$

Symbolic calculations in Matlab verify that $\frac{\partial \Gamma}{\partial x}$ is nonsingular over the operating envelope. The four flexible states η_i and $\dot{\eta}_i$ along with the pitch dynamics θ and Q become an unstable internal dynamics for the plant. While explicit calculation of the normal form in coordinates proved to be intractable, Jacobian linearization about a trim condition verifies that the internal dynamics are unstable. It should be emphasized that the pitch dynamics become unstable under feedback linearization, even if the flexible effects are removed from the system. Since the presence of unstable internal dynamics substantially complicates the controller design, an approach for removing the internal dynamics from the model used for control design will be adopted in the sequel.

III. Simplified Nominal Model

The six dimensional, unstable internal dynamics makes the nominal model unsuitable for feedback linearization control. To avoid dealing with the internal dynamics directly, a *simplified nominal model* which enjoys the property of full vector relative degree will be derived.

Despite their complex coupling to the rest of the system, the flexible modes are asymptotically stable with a reasonably large damping ratio. Thus, in our attempt to create a tractable model, the flexible states will be removed from the simplified nominal model. In addition, the coupling of the elevator deflection δ_e to the L and D forces is an undesirable parasitic effect. By including this term, δ_e appears in lower order derivatives of V_t and h, requiring additional dynamic extension to achieve vector relative degree. A very similar situation involving parasitic couplings in vertical take-off and landing (VTOL) aircraft is discussed by Hauser et al. in Ref. 9. By ignoring the weak elevator couplings, in particular setting to zero the terms $C_L^{\delta_e}$, $C_D^{\delta_e}$, and $C_D^{\delta_e^2}$, the second order dynamic extension of δ_e becomes unnecessary. Figure 2 shows the effect of neglecting these couplings on the curve fits for L and D. While still present, these effects are clearly dominated by the terms depending on α.



(a) Drag Force with and without δ_e coupling

(b) Lift Force with and without δ_e coupling

Figure 2. Neglecting the coupling of δ_e to L and D does not significantly alter the accuracy of the fit.

With the removal of the four flexible states and the parasitic elevator couplings, the simplified nominal model has only seven states: the five original aerodynamic variables and the two integrators appended to the Φ dynamics from (5). The inputs for this version of the model are Φ_c and δ_e. Symbolic calculations reveal that the resulting model has full vector relative degree $r = 7$. The following section discusses the control design based on this model.

IV. Approximate Feedback Linearization Design

The feedback linearization design for the simplified nominal model was completed using a generic feedback linearization code for square MIMO systems that was developed using the Matlab symbolic toolbox. Since the system has full relative degree, this design process proceeds using standard techniques.[10,13] For simplicity, denote the state of the simplified nominal model as $x \in \mathbb{R}^7$. The system can then be rewritten in normal form using a suitable change of coordinates to obtain

$$
\begin{aligned}
z = \Gamma(x)\,, \qquad \Gamma(x) &= \begin{bmatrix} V_t \\ L_f V_t \\ L_f^2 V_t \\ h \\ L_f h \\ L_f^2 h \\ L_f^3 h \end{bmatrix} \\
\dot{z}_1 &= z_2 \\
\dot{z}_2 &= z_3 \\
\dot{z}_3 &= L_f^3 V_t + (L_{\Phi_c} L_f^2 V)\Phi_c + (L_{\delta_e} L_f^2 V)\delta_e
\end{aligned}
$$

$$
\begin{aligned}
\dot{z}_4 &= z_5 \\
\dot{z}_5 &= z_6 \\
\dot{z}_6 &= z_7 \\
\dot{z}_7 &= L_f^4 h + (L_{\Phi_c} L_f^3 h)\Phi_c + (L_{\delta_e} L_f^3 h)\delta_e.
\end{aligned}
$$

Denoting the outer loop controls by u_V and u_h, the following control law renders the input/output map linear

$$
\begin{aligned}
\begin{bmatrix} \Phi_c \\ \delta_e \end{bmatrix} &= A^{-1} \begin{bmatrix} u_V - L_f^3 V \\ u_h - L_f^4 h \end{bmatrix} \\
A(x) &= \begin{bmatrix} L_{\Phi_c} L_f^2 V & L_{\delta_e} L_f^2 V \\ L_{\Phi_c} L_f^3 h & L_{\delta_e} L_f^3 h \end{bmatrix}.
\end{aligned}
$$

Specifically, the system under this control can be rewritten as

$$
\begin{aligned}
V_t^{(3)} &= u_V \\
h^{(4)} &= u_h.
\end{aligned}
$$

The outer loop linear controllers for these two decoupled integrator chains are designed using LQR with integral augmentation and a model reference. The LQR design weights are selected as

$$
\begin{aligned}
Q_{V_t} &= \mathrm{diag}(10, 1, 1, 1) \\
R_{V_t} &= 1 \\
Q_h &= \mathrm{diag}(10, 1, 1, 1, 1) \\
R_h &= 1.
\end{aligned}
$$

To obtain a reference model for each linear system to track, the reference signals for V_t and h are filtered using fast low pass filters with unity DC gain. The outputs of the reference model are denoted V_{ref} and h_{ref}. By using a filter of sufficiently high order for each signal, the derivatives of the reference signals used below are made available to the outer loop controller. Specifically, denoting with K_V and K_h the LQR design gains, the outer loop controller is given by

$$
\begin{aligned}
u_V(t) &= V_{\mathrm{ref}}^{(3)} - K_V \begin{bmatrix} V - V_{\mathrm{ref}} \\ L_f V - \dot{V}_{\mathrm{ref}} \\ L_f^2 V - \ddot{V}_{\mathrm{ref}} \\ \int_0^t (V(\tau) - V_{\mathrm{ref}}(\tau)) d\tau \end{bmatrix} \\
u_h(t) &= h_{\mathrm{ref}}^{(4)} - K_h \begin{bmatrix} h - h_{\mathrm{ref}} \\ L_f h - \dot{h}_{\mathrm{ref}} \\ L_f^2 h - \ddot{h}_{\mathrm{ref}} \\ L_f^3 h - h_{\mathrm{ref}}^{(3)} \\ \int_0^t (h(\tau) - h_{\mathrm{ref}}(\tau)) d\tau \end{bmatrix}.
\end{aligned}
$$

Figure 3 provides a conceptual diagram of the completed controller.

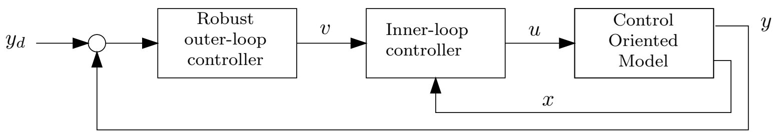


Figure 3. The inner loop controller represents the approximate feedback linearization controller design based on the simplified nominal model, while the robust outer loop controller represents the outer loop LQR.

The decoupling matrix $A(x)$ was verified to be non-singular over the operating range of interest. Symbolic calculations reveal that there are four real roots of the determinant of A. Two of the roots, $V = 0$ and

$\alpha = \theta + \frac{\pi}{2}$, correspond to non-physical situations and were encountered in other similar models.[20] One additional singular point corresponds to a an unattainable value of h. The final solution yields somewhat plausible values for α, but these values are all outside the anticipated operating envelope.

The controller is guaranteed by design to perform well on the simplified nominal system. Simulations verify that the controller linearizes the simplified nominal model up to the numerical accuracy of the algorithm used to integrate the nonlinear differential equations, provided that the variation of β_* is removed. (Recall from section II that the slow variation of β_* with V_t and h is neglected in the control design.) Even with this variation in place, the input/output map is very nearly linear. Simulation results on the nominal model demonstrate good tracking performance. These results will be omitted both for brevity and due to their similarity to the truth model results, which are presented in the next section.

V. Truth Model Simulation Results

The truth model is implemented in Simulink using S-functions.[2,8] All simulations used fixed step Runge-Kutta integration with a step size of 0.05 seconds. Since the truth model does not support simulation of high fuel to air ratios,[2] the attainable thrust is limited and all reference trajectories are passed through a rate limiter followed by a low pass filter of the form

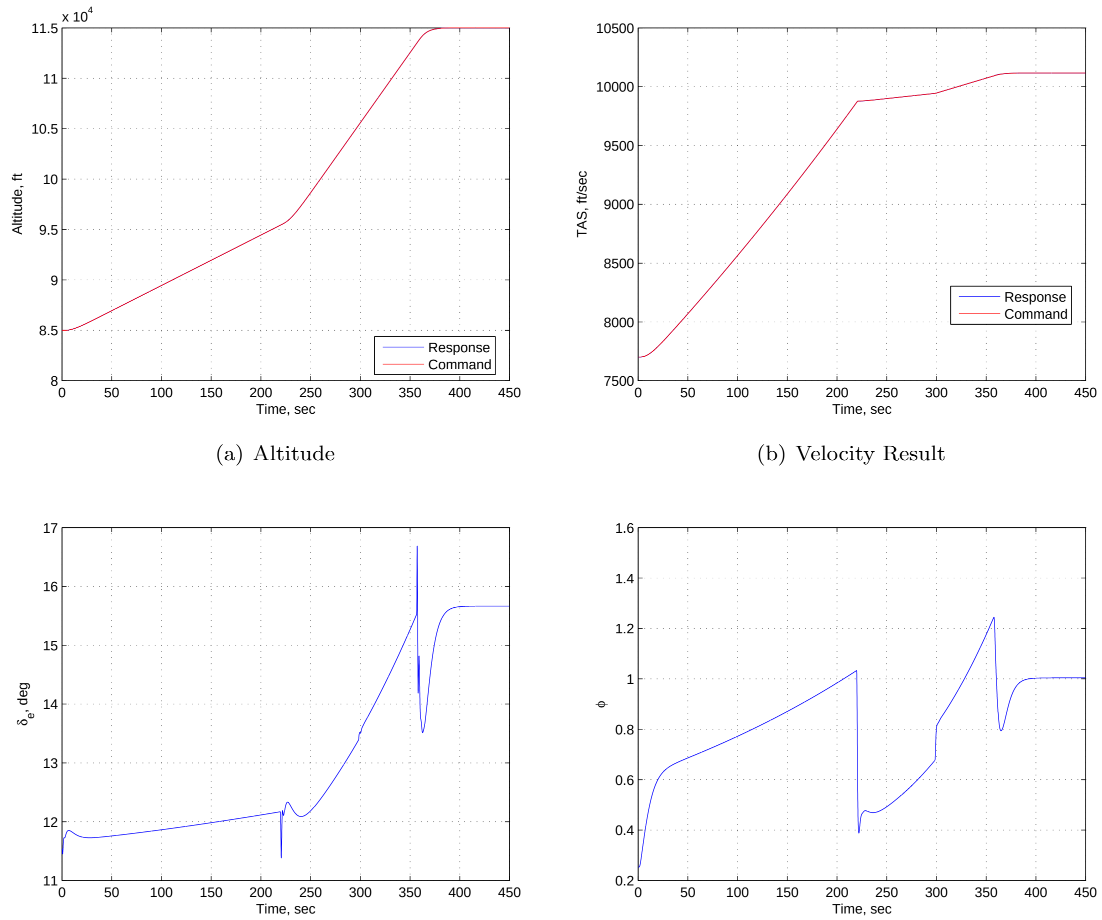
$$H(s) \quad = \quad \frac{.04}{s^2 + .4s + .04}.$$

The selected reference trajectory begins at $\bar{q} = 2000\frac{\text{lb}}{\text{ft}^2}$ and $h = 85000$ ft. The aircraft climbs at a steady $50\frac{\text{ft}}{\text{s}}$ while maintaining a constant dynamic pressure. Once the HSV reaches mach 10, the mach number is held constant and the climb rate increases to $139\frac{\text{ft}}{\text{s}}$ until levelling off at $115,000$ ft. This trajectory was determined to be a plausible operating trajectory for the vehicle, and the ramp increase in altitude provides a moderately aggressive tracking challenge to the controller. It should be noted that the available thrust limits the climb rate for this reference trajectory. If a faster climb rate is selected for the first leg, the required acceleration cannot be achieved without violating the actuator limits on Φ.[2]

Figures 4 and 5 show the simulation results for the test trajectory on the truth model under the proposed controller. The tracking performance is excellent throughout the entire maneuver, and both the flexible states and pitch dynamics remain stable and well-behaved. Simulations were also conducted with mild plant parameter variations to quantitatively assess the robustness of the controller. This brief case study revealed that the controller is at least moderately robust. Figure 6 gives an example result from this study in which the vehicle mass was increased by roughly 10%, I_{yy} was decreased by 25%, and the vehicle length was reduced by 15%. The tracking performance is still very good, the most noticeable difference being an overshoot in V_t when the trajectory reaches mach 10 and levels off.

VI. Control Design for an Alternative Output Choice

While the output choice V_t and h is quite logical for an auto-pilot, a controller designed for V_t and the flight path angle γ would be more logical for use by a pilot. With this output selection, the simplified nominal model has vector relative degree $r = 6$, the one dimensional internal dynamics becoming that of the vehicle altitude h. However, on the output-zeroing manifold $\gamma = 0$, and examination of equation 2 reveals that the zero dynamics are given by the trivial equation $\dot{h} = 0$. Thus the altitude will remain stable even though it is rendered unobservable by the linearizing feedback. Figure 7 and 8 show simulation results for the controller using V_t and γ as the regulated output. As expected, the trajectory of the altitude h remains bounded. It should be noted that the test trajectory used for this controller is a simple step increase in V_t and a unitary pulse for γ. Also, the outer loop controller was implemented using a simple pole placement design. A more sophisticated outer loop design similar to the one employed for the V_t and h output choice could yield superior performance for this choice of output.



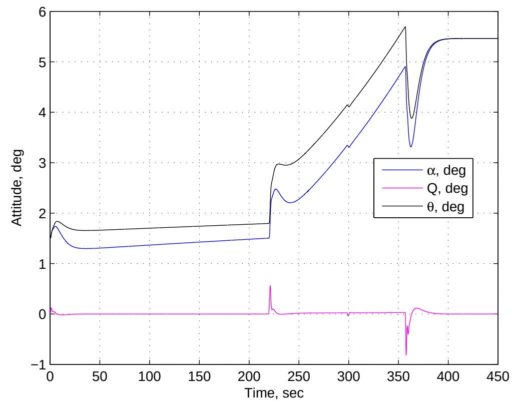
(a) Altitude

(b) Velocity Result

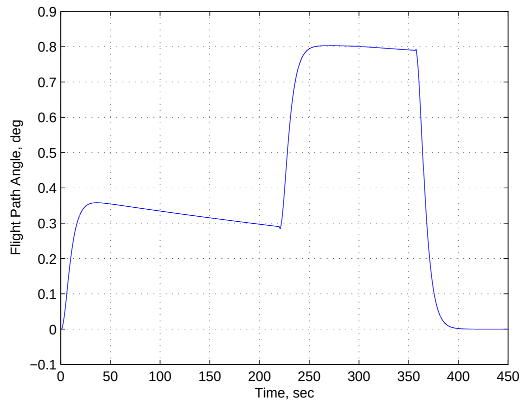
(c) δ_e

(d) Φ

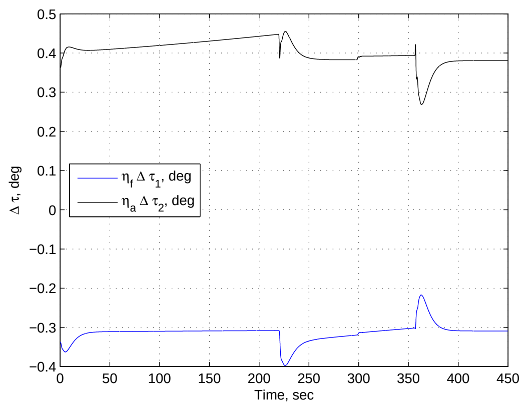
Figure 4. Simulation Result for Baseline Trajectory on the Truth Model



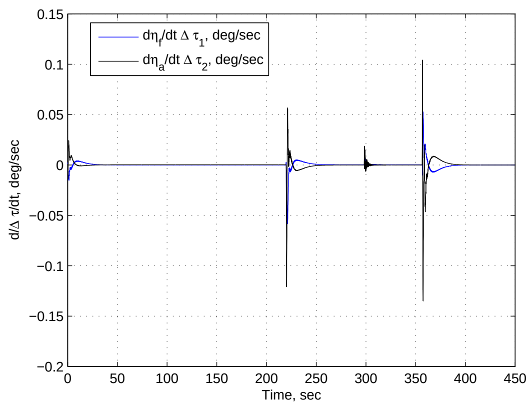
(a) Angular States



(b) Flight Path Angle

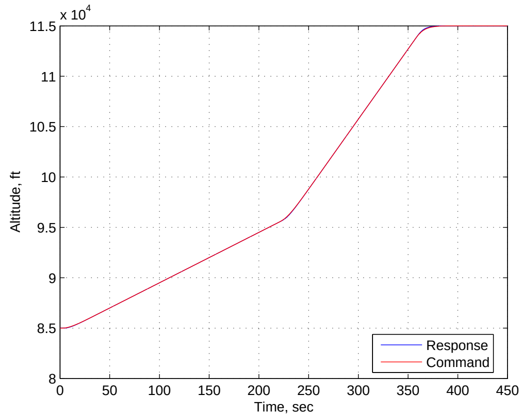


(c) Flexible States

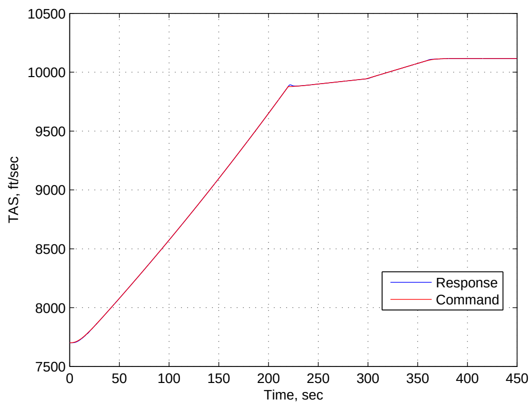


(d) Flexible State Rates

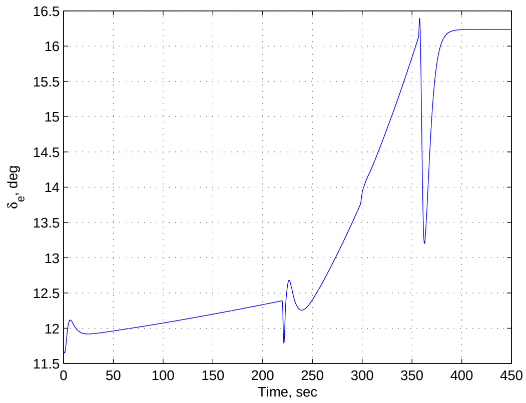
Figure 5. Simulation Result for Baseline Trajectory on the Truth Model



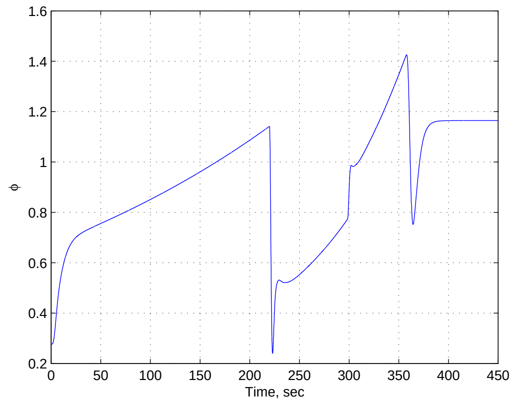
(a) Altitude



(b) Velocity Result

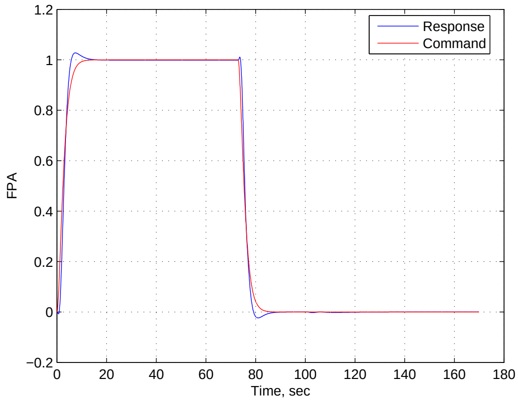


(c) δ_e

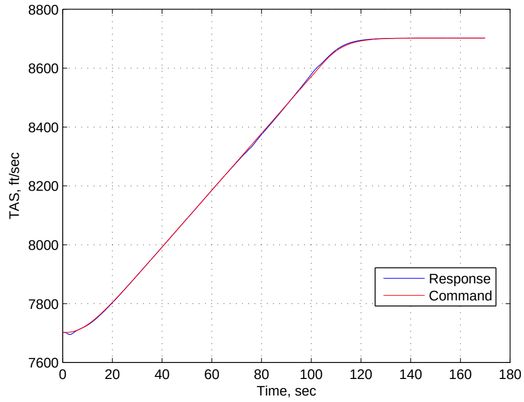


(d) Φ

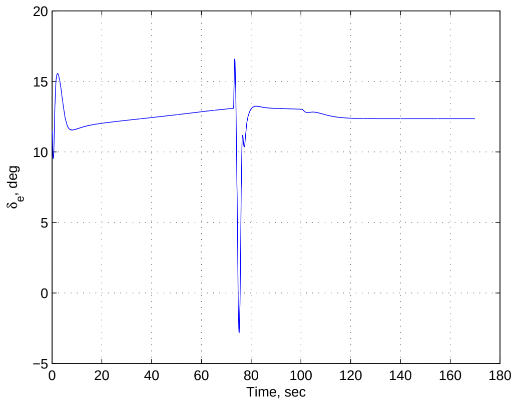
Figure 6. Simulation Result for Baseline Trajectory on the Truth Model with Parameter Variation.



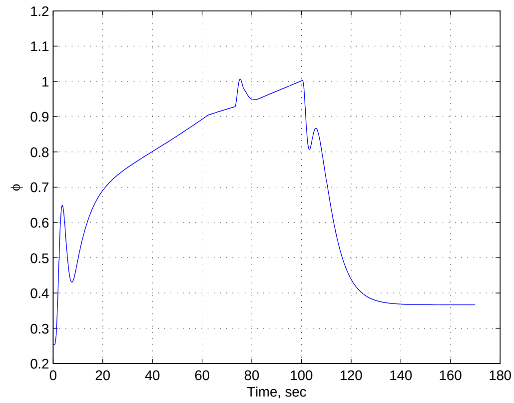
(a) Flight Path Angle



(b) Velocity Result

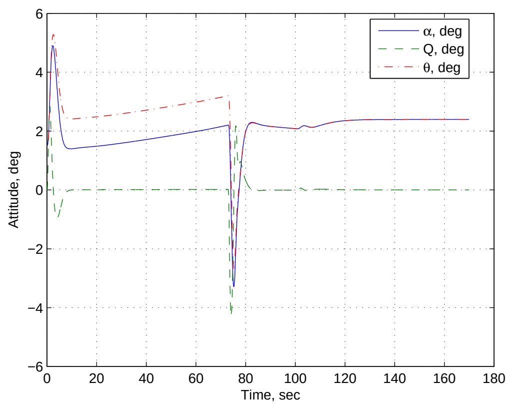


(c) δ_e

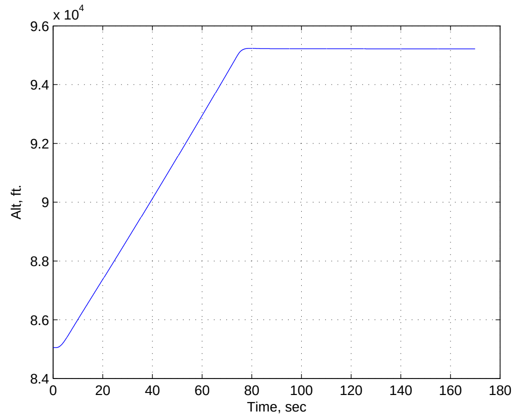


(d) Φ

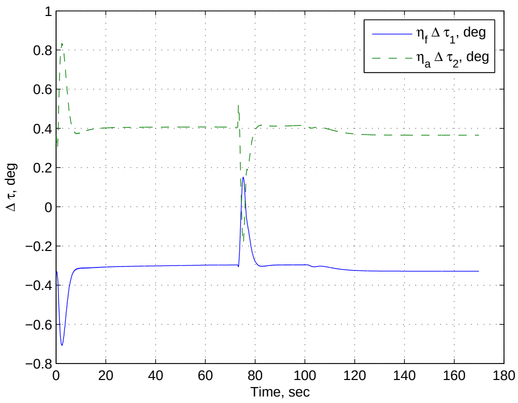
Figure 7. Simulation Result for γ and V Tracking



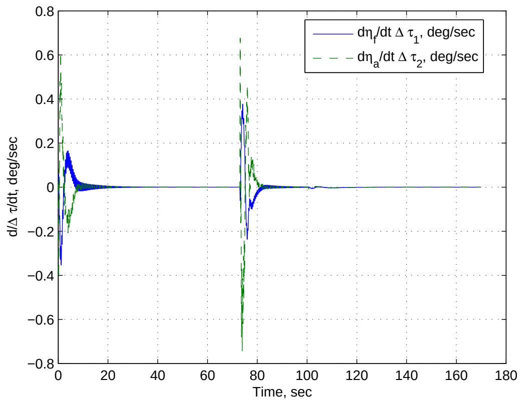
(a) Angular States



(b) Altitude



(c) Flexible States



(d) Flexible State Rates

Figure 8. Simulation Result for γ and V Tracking

VII. Conclusions

In this paper we have demonstrated the suitability of control system design based on approximate feedback linearization for an air-breathing hypersonic vehicle. The key steps in the design are the construction of an analytically tractable nominal model and the removal of certain flexible effects and weak couplings to achieve full relative degree for a simplified model. The resulting controller achieves comparable, if not superior, performance to earlier linear control designs[8] over a wider operating range. In addition, since this approach is more closely tied to the nonlinear model, an extensive tuning algorithm was not necessary to achieve excellent tracking performance. Of course, the penalty for this more straightforward outer loop design lies in the dramatically increased complexity of both the system identification process and the computational complexity of the controller itself.

The simplified nominal model was constructed in the spirit of approximate feedback linearization as proposed by Hauser et al.[9] Future work will include an analytical proof of stability for the nominal system under the designed control law, along with an attempt to generalize this result to a class of similar systems. In addition, more elaborate vehicle models with additional flexibility effects, thermal effects, and engine dynamics will be considered. Additional control effectors, such as the canard used in a recent paper by Bolender,[?] may also be employed to improve control performance for these more sophisticated models.

VIII. Acknowledgements

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